Induction and Inductance

Two experiments

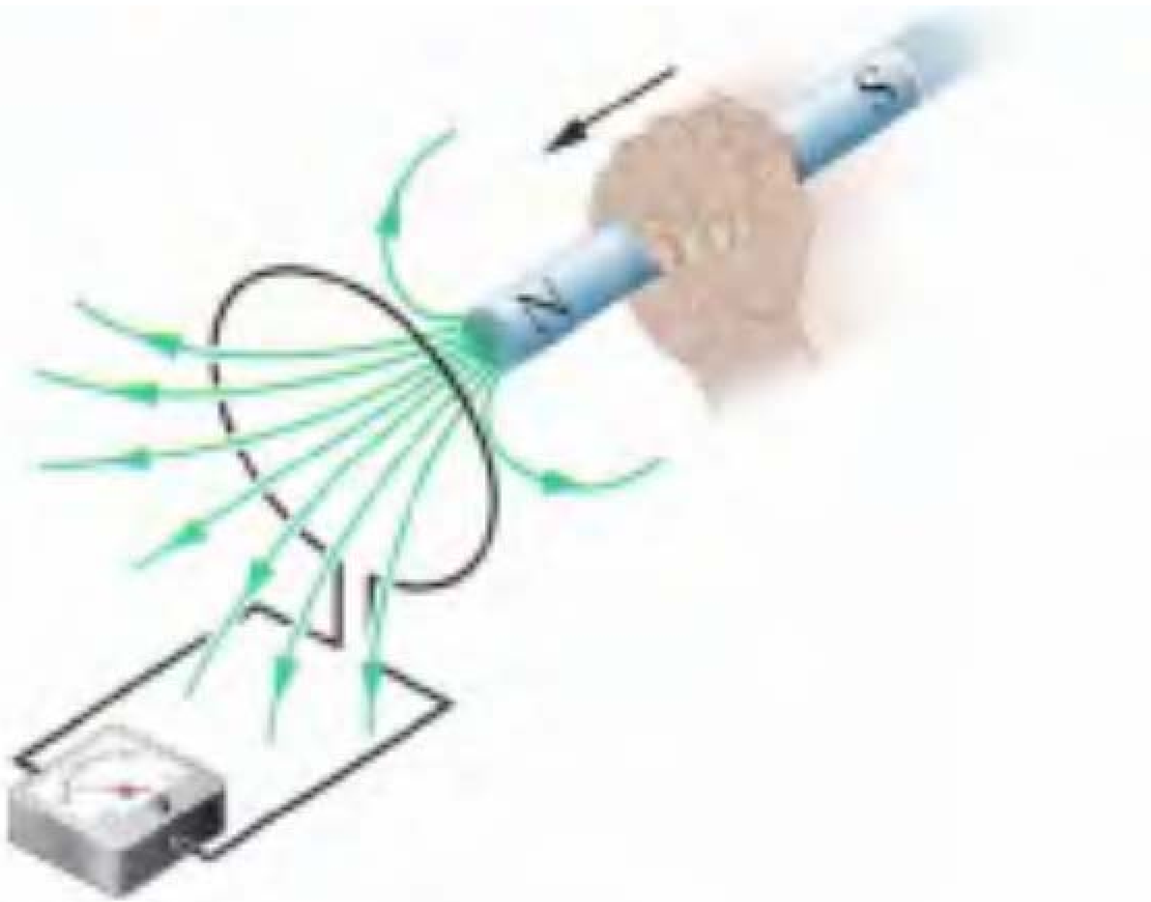


FIG. 30-1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

- Current only if motion
- Faster motion – more current
- Move in opposite direction, current in reverse direction

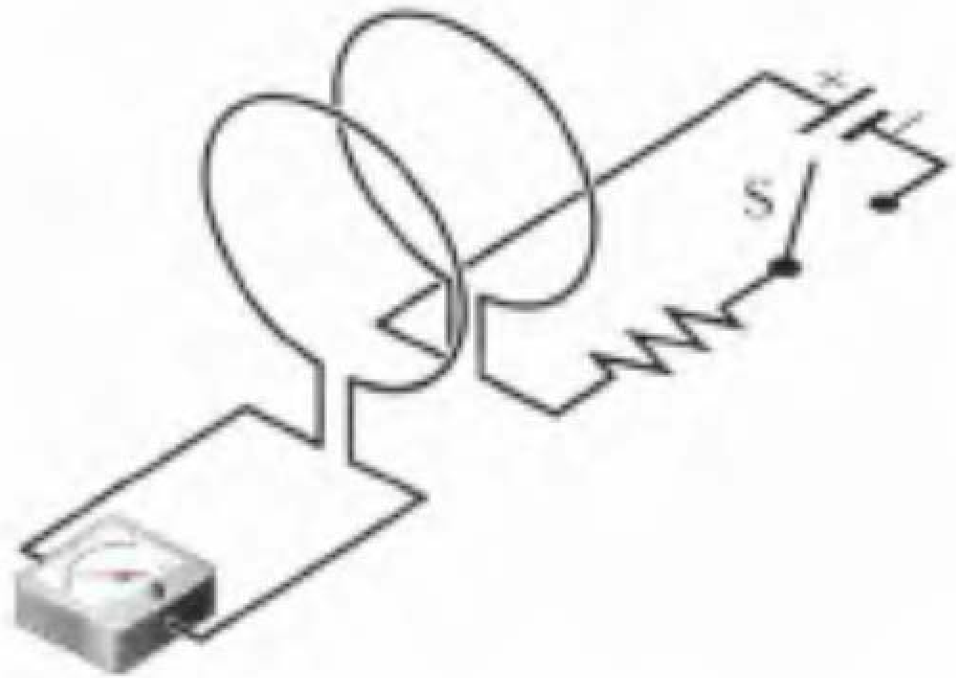


FIG. 30-2 An ammeter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right-hand wire loop) or opened (to turn off the current in the right-hand loop). No motion of the coils is involved.

Two experiments

- First shows that a moving magnet induces a current
- Second shows that a changing current produces a current in the second loop.

Faraday's law of Induction

- An emf is induced in a loop when the number of magnetic field lines ("magnetic flux") that pass thru the loop is changing

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \qquad \text{(magnetic flux through area } A\text{).}$$

Units: tesla square meter = Weber = Wb

If perpendicular and uniform:

$$\Phi_B = BA \qquad (\vec{B} \perp \text{area } A,\ \vec{B} \text{ uniform}).$$

Faraday's law of Induction

The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} \qquad \text{(coil of } N \text{ turns).}$$

Can generate emf by:

- Changing B
- Changing area of affected part of coil
- Changing angle between B and plane of coil

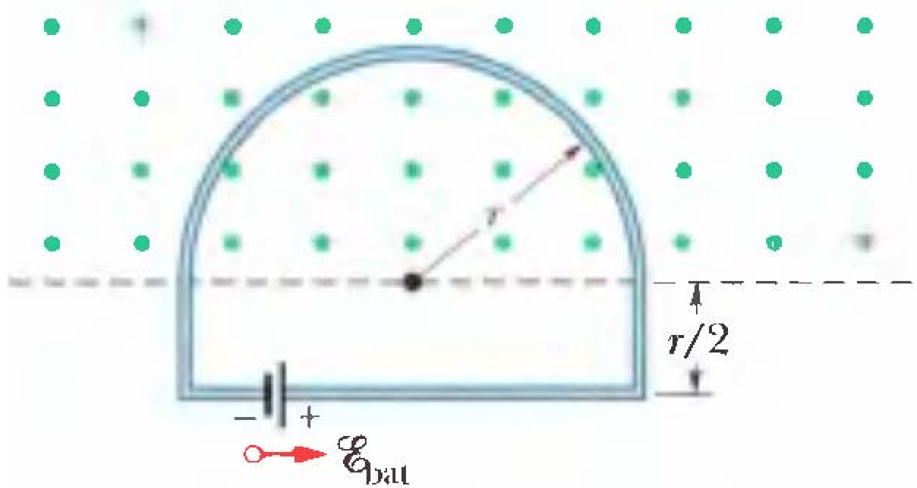
Lenz's Law

An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

Figure 30-8 shows a conducting loop consisting of a half-circle of radius $r = 0.20$ m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\mathscr{E}_{bat} = 2.0$ V is connected to the loop. The resistance of the loop is 2.0 Ω.

(a) What are the magnitude and direction of the emf $\mathscr{E}_{ind}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

(b) What is the current in the loop at $t = 10$ s?



A current thru a wound coil produces a magnetic flux in its center

Define inductance as:

$$L = \frac{N\Phi_B}{i} \qquad \text{(inductance defined)},$$

Unit: $1 \text{ henry} = 1 \text{ H} = 1 \text{ T} \cdot \text{m}^2/\text{A}.$

(skip derivation)

Induction is a property of the coil

Self-Induction

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.

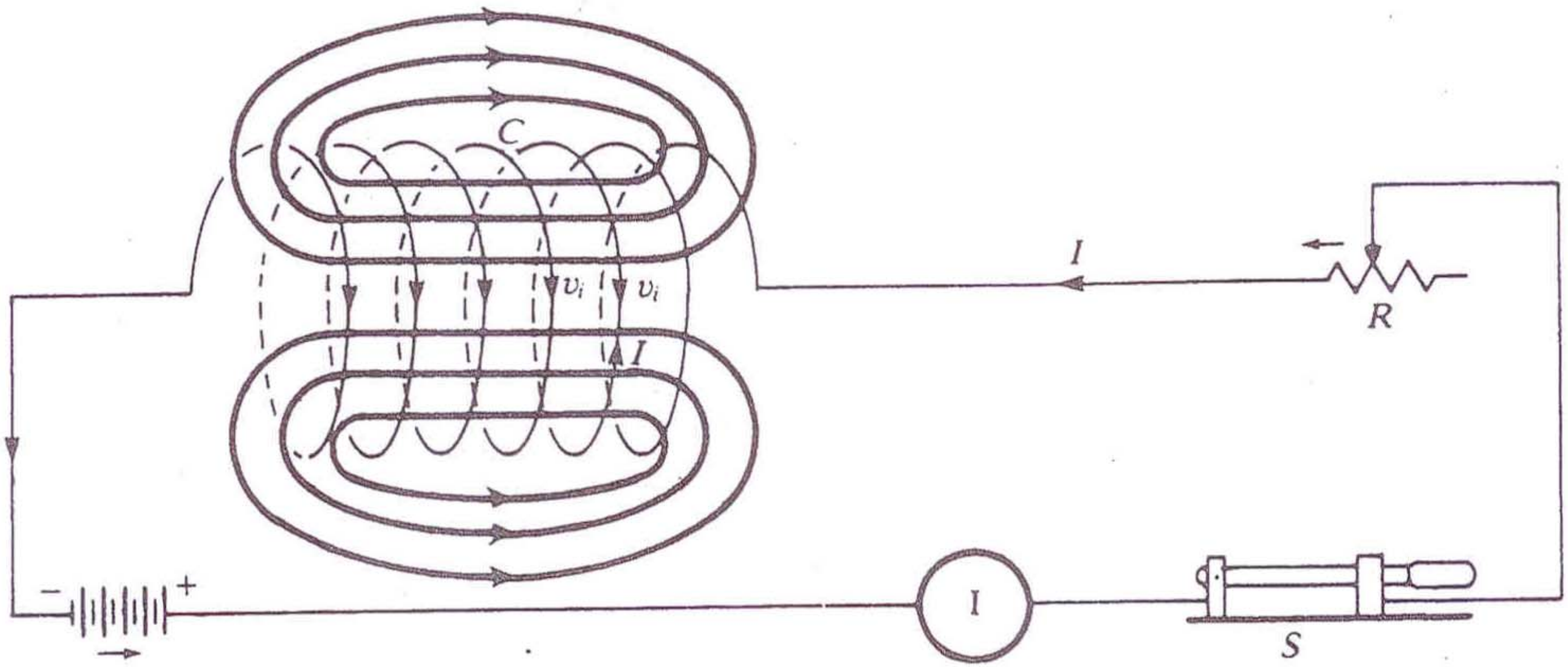
A changing current in a loop causing a changing magnetic field which induces an emf

$$N\Phi_B = Li.$$

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

$$\mathscr{E}_L = -L\frac{di}{dt} \qquad \text{(self-induced emf).}$$

Will be in the direction to OPPOSE change in i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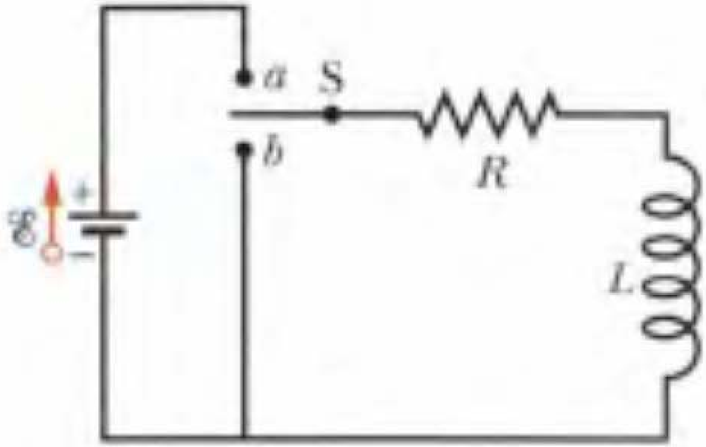
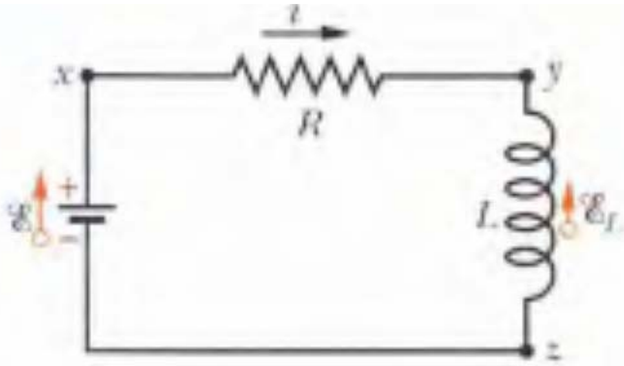




$$-iR - L\frac{di}{dt} + \mathscr{E} = 0$$

$$L\frac{di}{dt} + Ri = \mathscr{E} \qquad (RL \text{ circuit}).$$

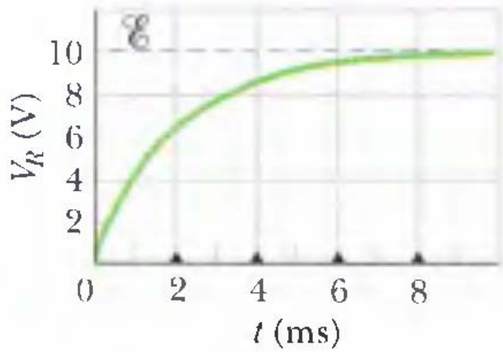
Looks similar to RC circuit with: I instead of q; L instead of R and R instead of 1/C

$$i = \frac{\mathscr{E}}{R}(1 - e^{-Rt/L}),$$

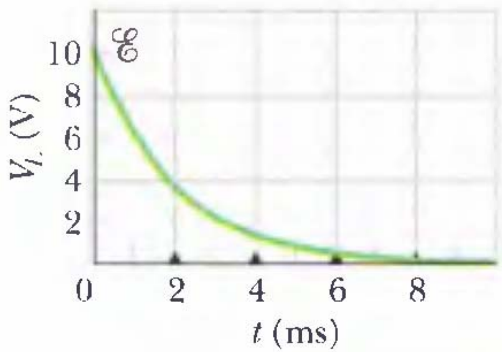
$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) \qquad \text{(rise of current).}$$

$$\tau_L = \frac{L}{R} \qquad \text{(time constant).}$$

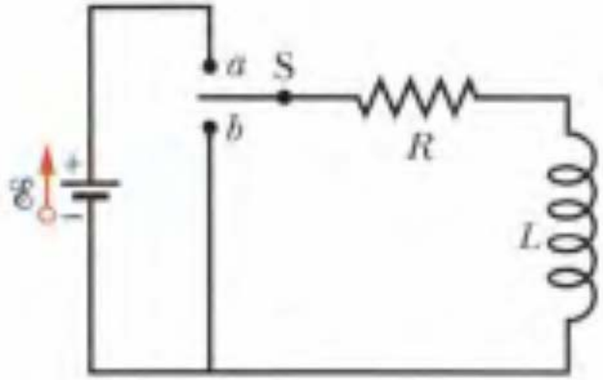
$$1\,\frac{\text{H}}{\Omega} = 1\,\frac{\text{H}}{\Omega}\left(\frac{1\text{ V}\cdot\text{s}}{1\text{ H}\cdot\text{A}}\right)\left(\frac{1\,\Omega\cdot\text{A}}{1\text{ V}}\right) = 1\text{ s}.$$



(a)



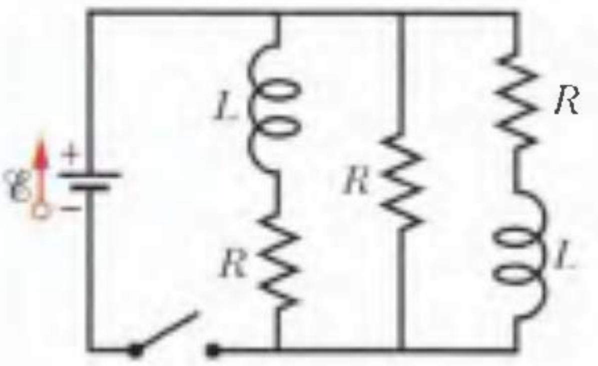
(b)



Flip switch to "b"

$$L\frac{di}{dt} + iR = 0.$$

$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L} \qquad \text{(decay of current)}.$$



(a)

Figure 30-23*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

(a) What is the current i through the battery just after the switch is closed?

(b) What is the current i through the battery long after the switch has been closed?

A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω. If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value?

Energy Stored in a Magnetic Field

Remember: $$\mathscr{E} = L\frac{di}{dt} + iR,$$

Multiply everything by "i": $$\mathscr{E}i = Li\frac{di}{dt} + i^2R,$$

Three terms are:

1) Rate at which battery does work $(\mathscr{E}\,dq)/dt$, or $\mathscr{E}i$.

3) Energy lost as heat in resistor

2) rate at which energy stored in magnetic field

$$\frac{dU_B}{dt} = Li\,\frac{di}{dt}.$$

$$dU_B = Li\,di.$$

$$\int_0^{U_B} dU_B = \int_0^{i} Li\,di$$

$$U_B = \tfrac{1}{2}Li^2 \qquad \text{(magnetic energy)},$$

A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

Mutual Induction