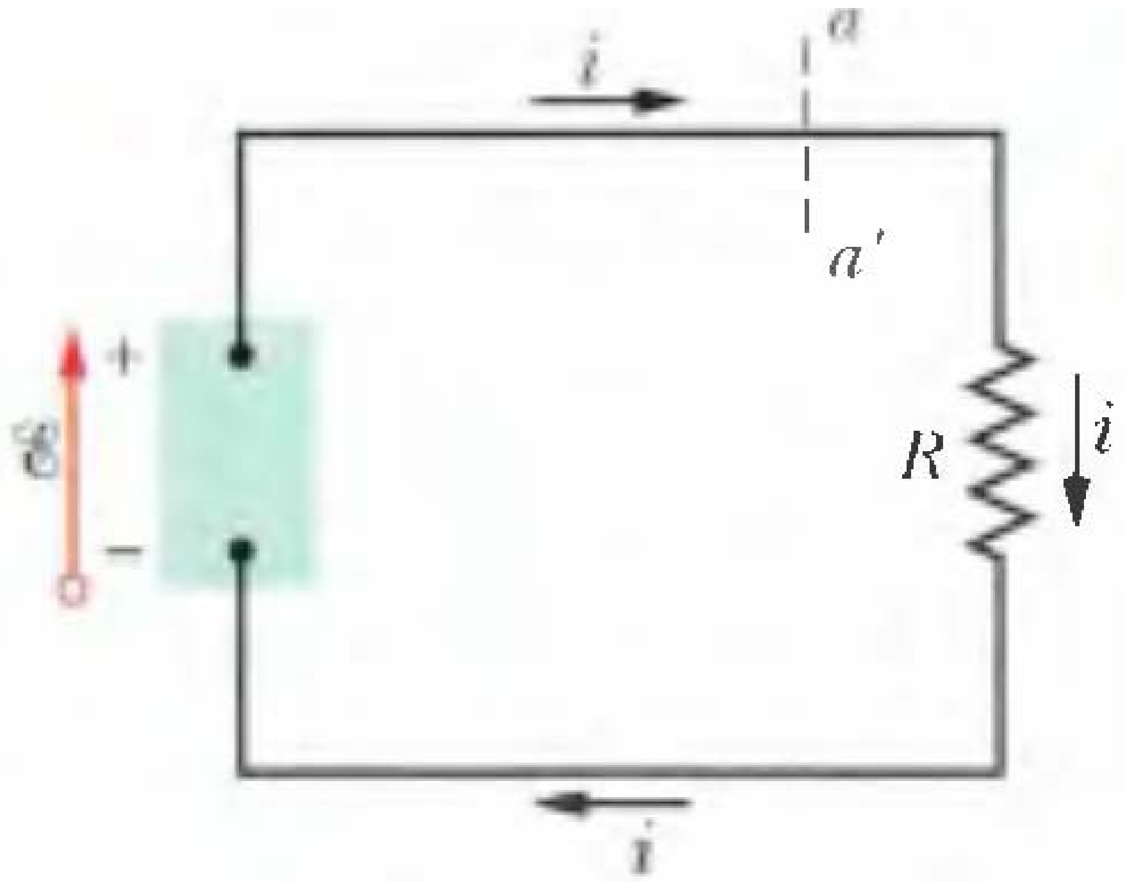
Circuits

Need “charge pump” – will call it an emf device – electromotive force -

Circuits

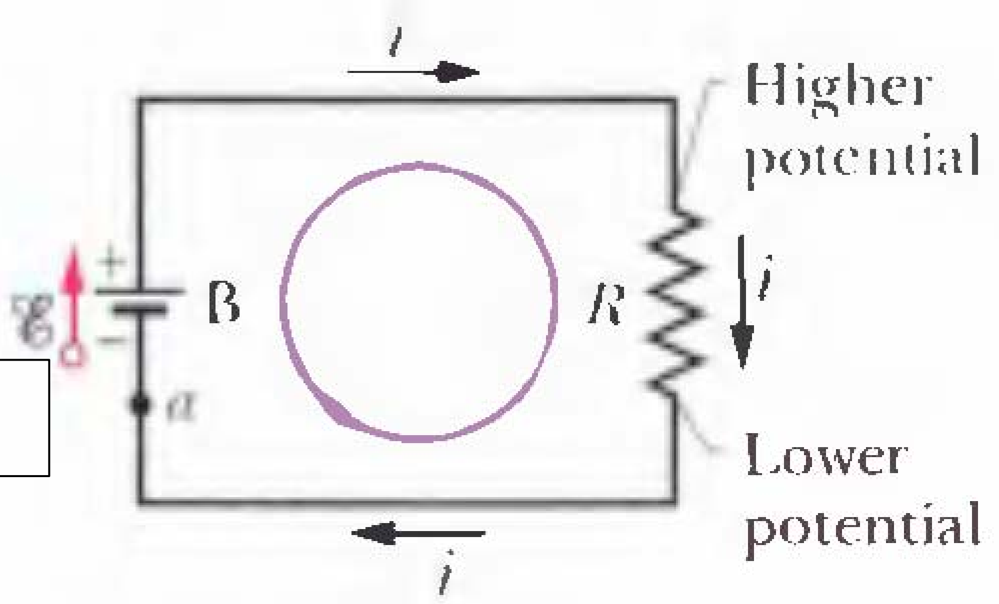


Kirchhoff's Voltage ("loop") Law - KVL

Sum of all changes in potential in any complete loop must be zero

Start at "a" and go around the loop

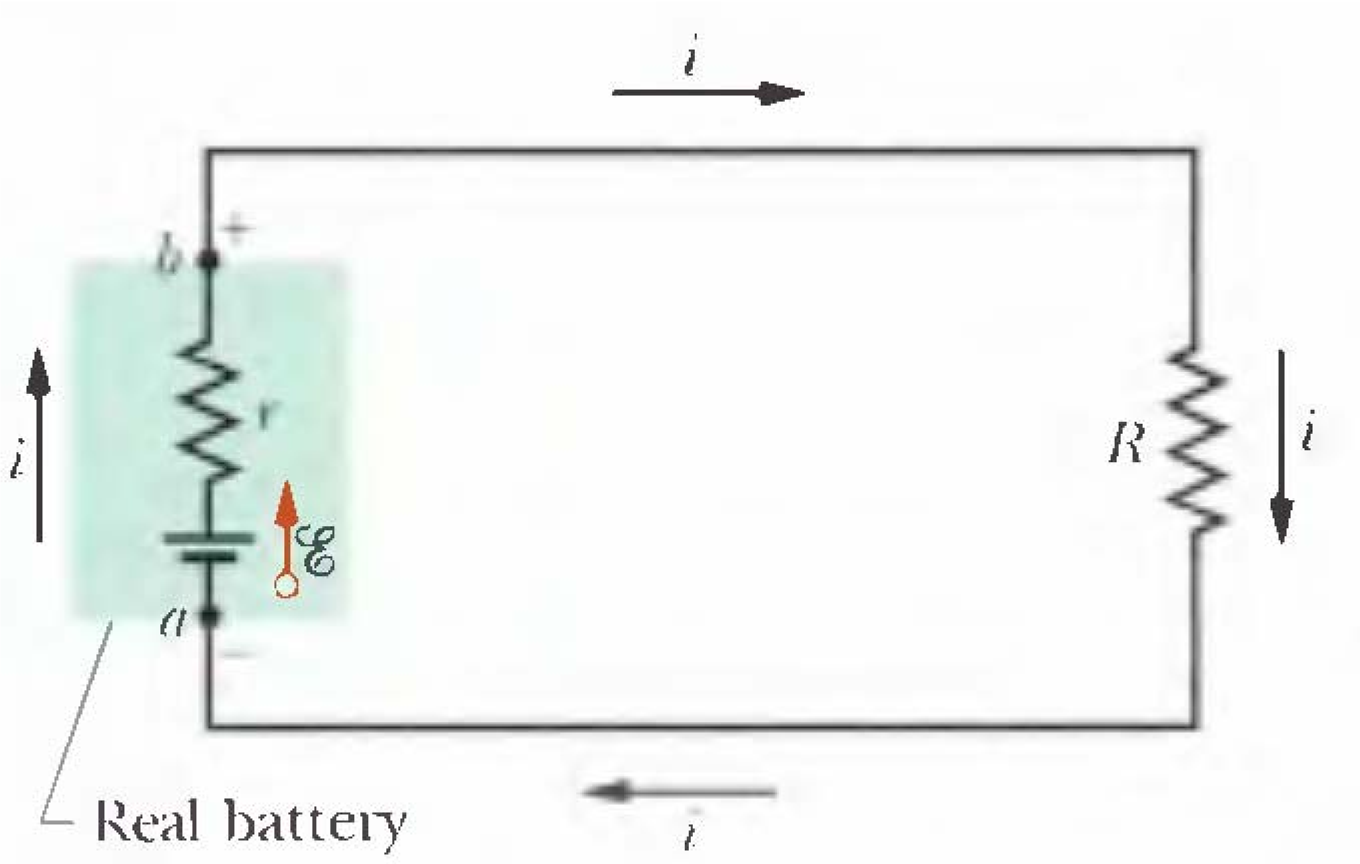
$$\mathcal{E} - iR = 0.$$



RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

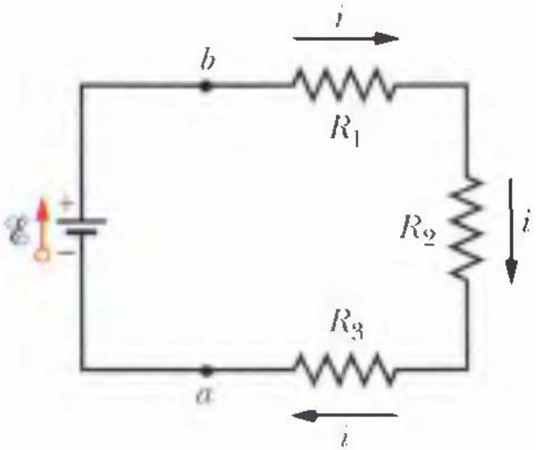
EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

Internal resistance in battery



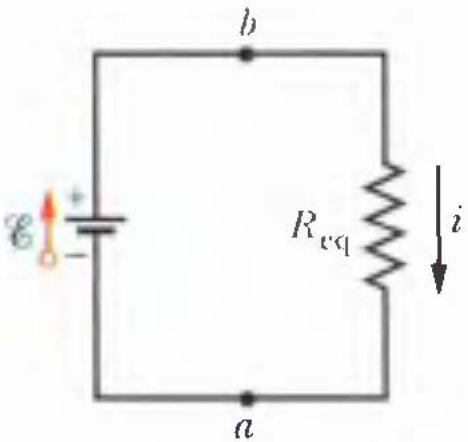
$$\mathscr{E} - ir - iR = 0.$$

Resistances in Series



$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0,$$

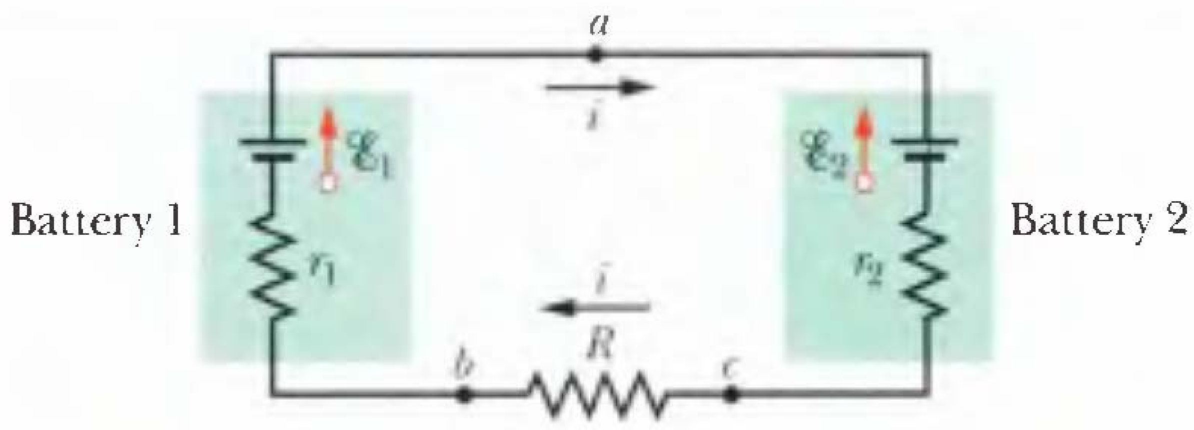
$$i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}.$$



$$\mathscr{E} - iR_{eq} = 0,$$

$$i = \frac{\mathscr{E}}{R_{eq}}.$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3.}$$



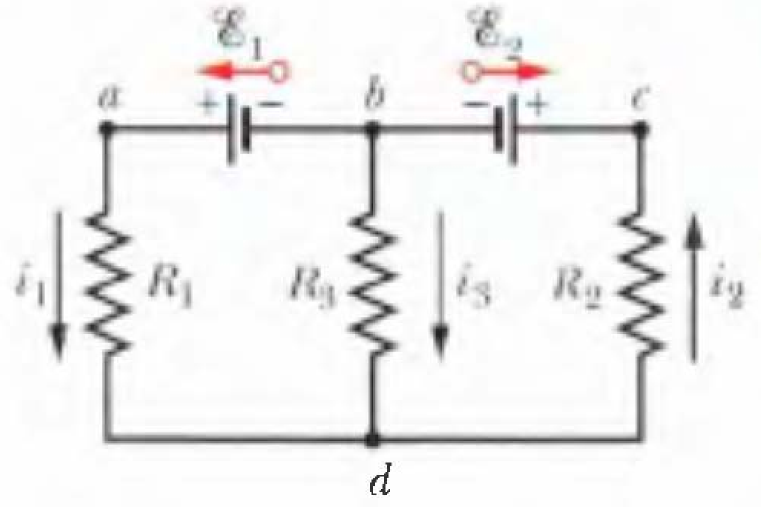
The emfs and resistances in the circuit of Fig. 27-8*a* have the following values:

$$\mathscr{E}_1 = 4.4\ \text{V}, \quad \mathscr{E}_2 = 2.1\ \text{V},$$
$$r_1 = 2.3\ \Omega, \quad r_2 = 1.8\ \Omega, \quad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit?

Kirchhoff's Current ("junction") Law - KCL

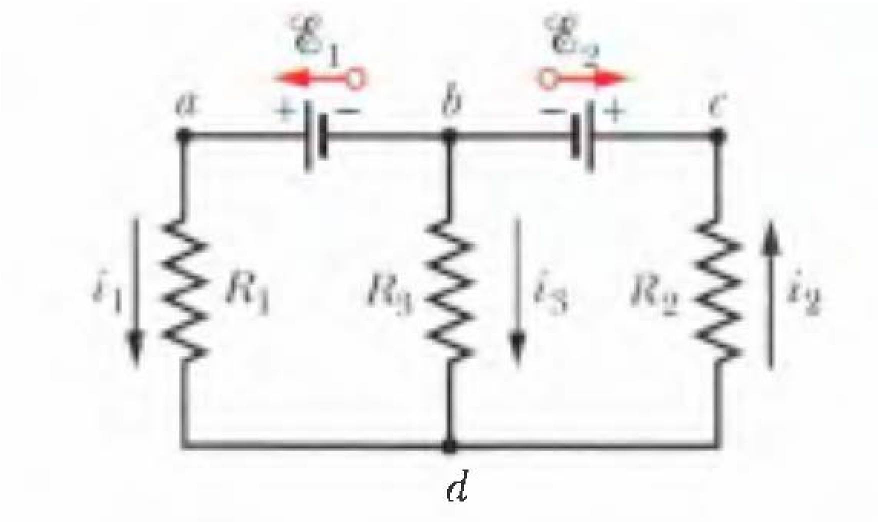
Multiloop Circuits



At node (junction) d

$$i_1 + i_3 = i_2.$$

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.



Loop equations

$$\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$$

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$$

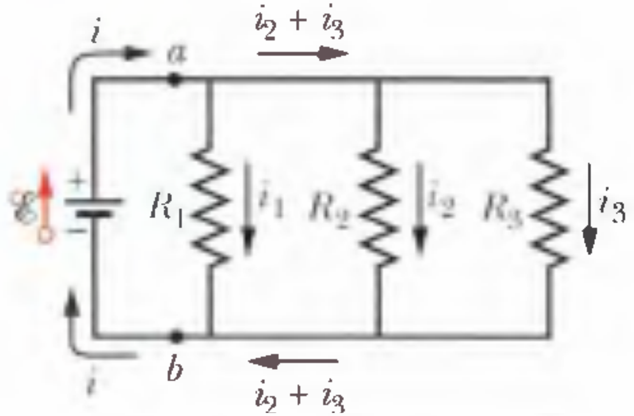
$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

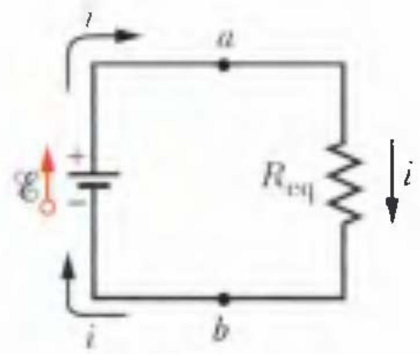
Note:

KVL is conservation of energy

KCL is conservation of charge

Resistances in Parallel





$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

$$i = \frac{V}{R_{eq}}.$$

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

$$i = \frac{V}{R_{eq}}.$$

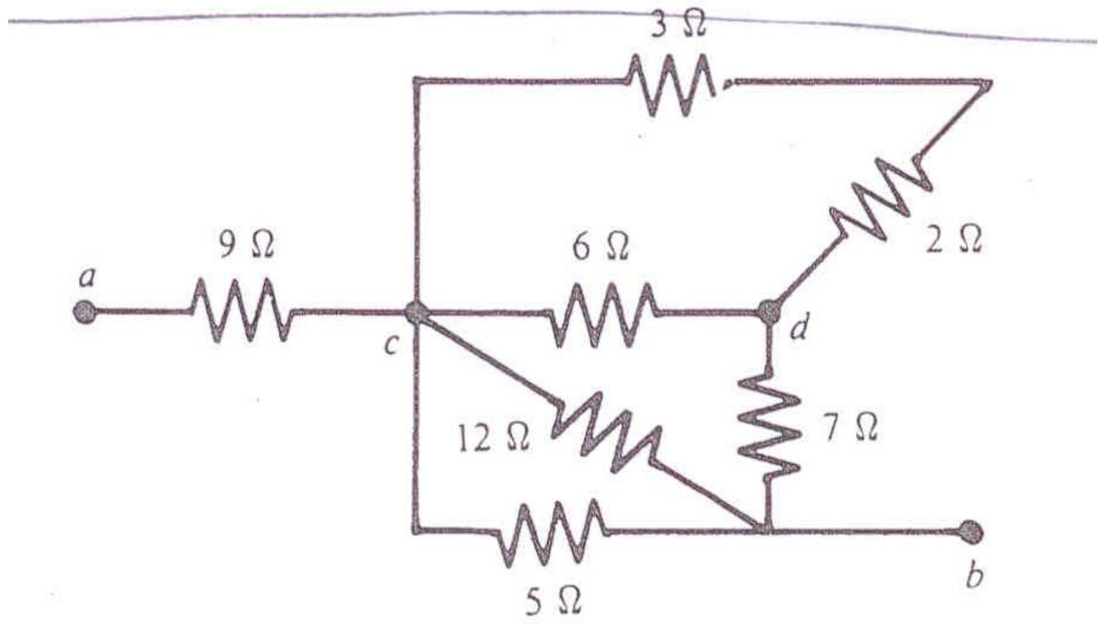
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.}$$

For 2 resistors: $$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$

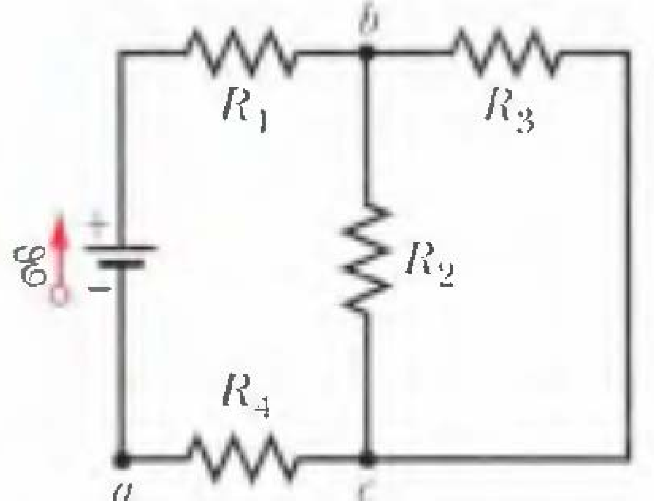
Series and Parallel Resistors and Capacitors

Series	Parallel
Resistors	
$R_{eq} = \sum_{j=1}^{n} R_j$	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$
Same current through all resistors	Same potential difference across all resistors

Series	Parallel
Capacitors	
$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$	$C_{eq} = \sum_{j=1}^{n} C_j$
Same charge on all capacitors	Same potential difference across all capacitors



Find the equivalent resistance between points a and b.



$R_1 = 20\ \Omega, \quad R_2 = 20\ \Omega, \quad \mathscr{E} = 12\ \text{V},$

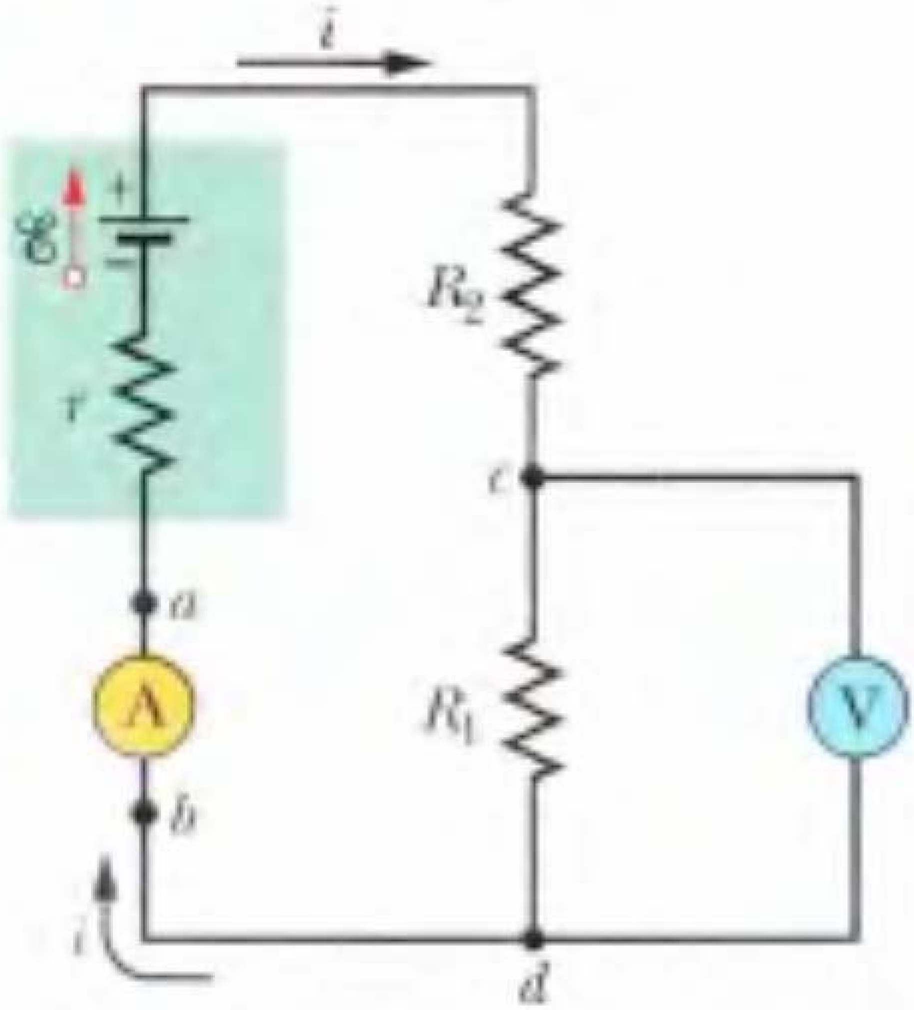
$R_3 = 30\ \Omega, \quad R_4 = 8.0\ \Omega.$

(a) What is the current through the battery?

(b) What is the current i_2 through R_2?

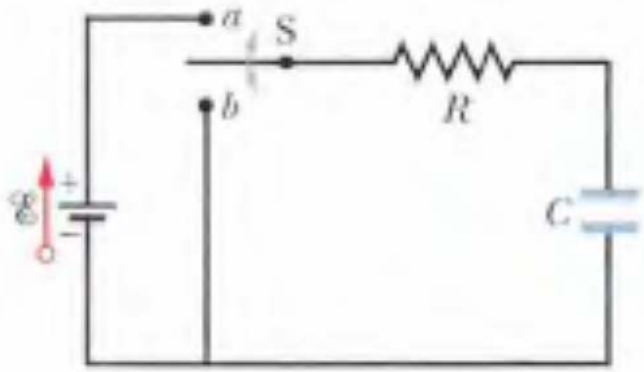
(c) What is the current i_3 through R_3?

The Ammeter and the Voltmeter



RC Circuits

Charging a Capacitor

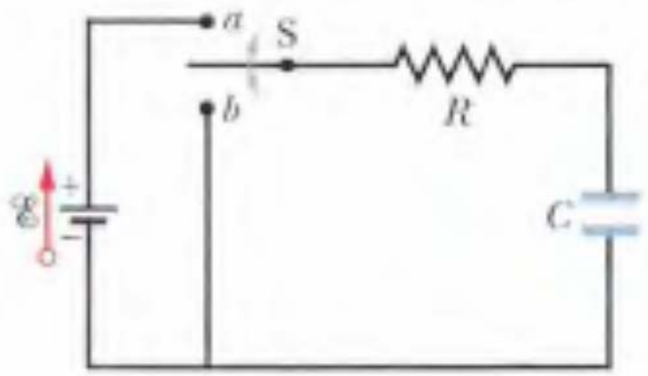


What happened when the switch is closed?

Charge starts loading onto the capacitor until q=cv and V=E

How does q, V, and i vary with time?
Use loop rule!

$$\mathscr{E} - iR - \frac{q}{C} = 0.$$



$$\mathscr{E} - iR - \frac{q}{C} = 0.$$

$$i = \frac{dq}{dt}.$$

$$R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E}$$

$$q = C\mathscr{E}(1 - e^{-t/RC})$$

At t=0, q=0

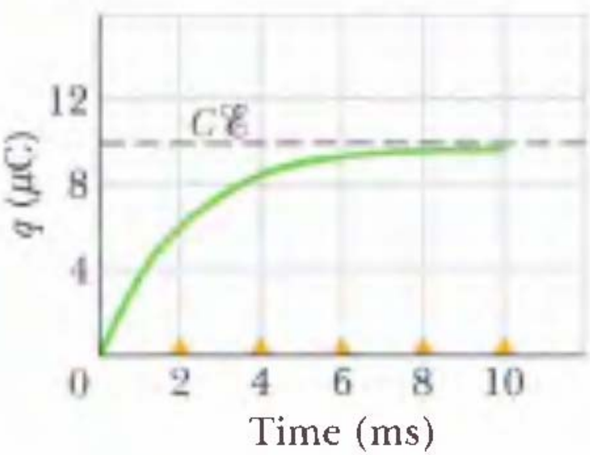
At t= infinity, q=CE

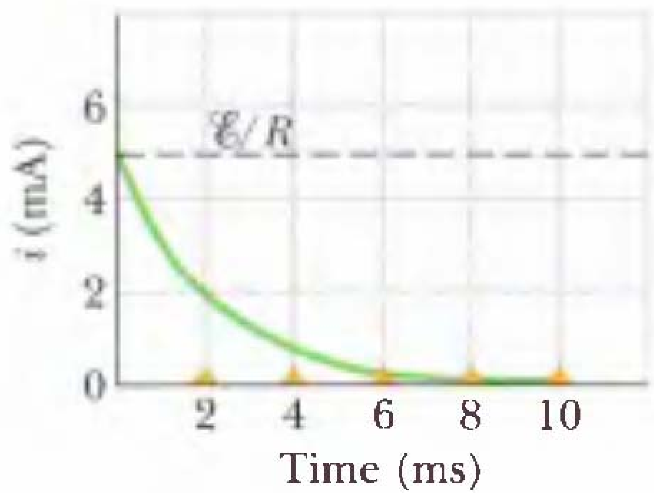
$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC}$$

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC})$$

$$q = C\mathscr{E}(1 - e^{-t/RC})$$

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC}$$





$$\tau = RC \qquad \text{(time constant).}$$

When t= $\tau = RC$

$$q = C\mathscr{E}(1 - e^{-1}) = 0.63C\mathscr{E}.$$

Discharging a Capacitor

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$q = q_0 e^{-t/RC}$$

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$

A camera flash gets its energy from a 150-μF capacitor and requires 170 V to fire. If the capacitor is charged by a 200-V source through an 18-kΩ resistor, how long must the photographer wait between flashes? Assume the capacitor is fully discharged with each flash.

Wheatstone Bridge

Describe a *Wheatstone bridge* and show how it can be used to find an unknown resistance X. What are the attractive features of measuring resistance this way?

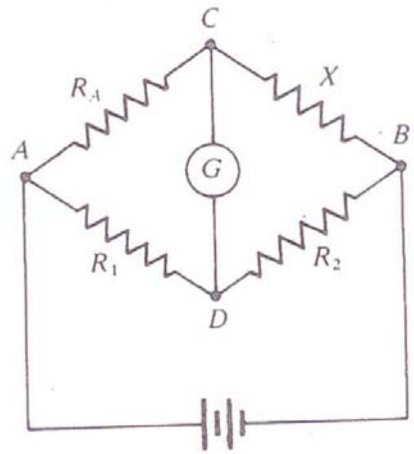


Fig. 27-24

A *Wheatstone bridge* is the circuit of Fig. 27-24. The terms R_A, R_1, and R_2 represent accurately known variable resistors whose values are set so that the galvanometer G reads zero. Then, points C and D are at the same potential, and the current through the upper branch is I_C and through the lower branch, I_D. Using these facts we have $I_C R_A = I_D R_1$, and $I_C X = I_D R_2$. To find X, we divide the second equation by the first:

$$\frac{X}{R_A} = \frac{R_2}{R_1} \qquad X = \frac{R_2}{R_1} R_A$$

Note that since the galvanometer is being used as a "null" instrument, we have no errors associated with the resistance of the instrument. Similarly, the actual terminal voltage of the battery need not be known, so no errors from that measurement enter.

The Wheatstone bridge shown in Fig. 27-25 is being used to measure resistance X. At balance, the current through the galvanometer G is zero and resistances L, M, and N are 3, 2, and 10 Ω, respectively. Find the value of X.