Capacitance

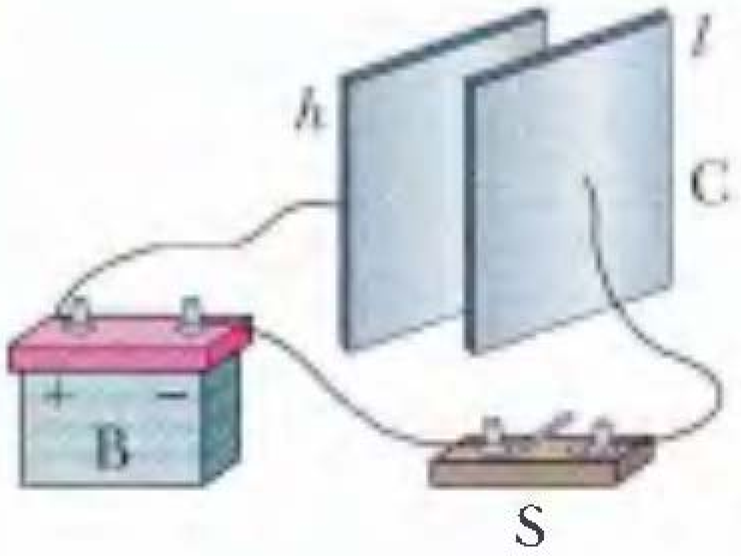
- Two isolated conductors
- Charge to equal and opposite charge
- Each is an equipotential surface
- Potential between them is such that:

$$q = CV.$$

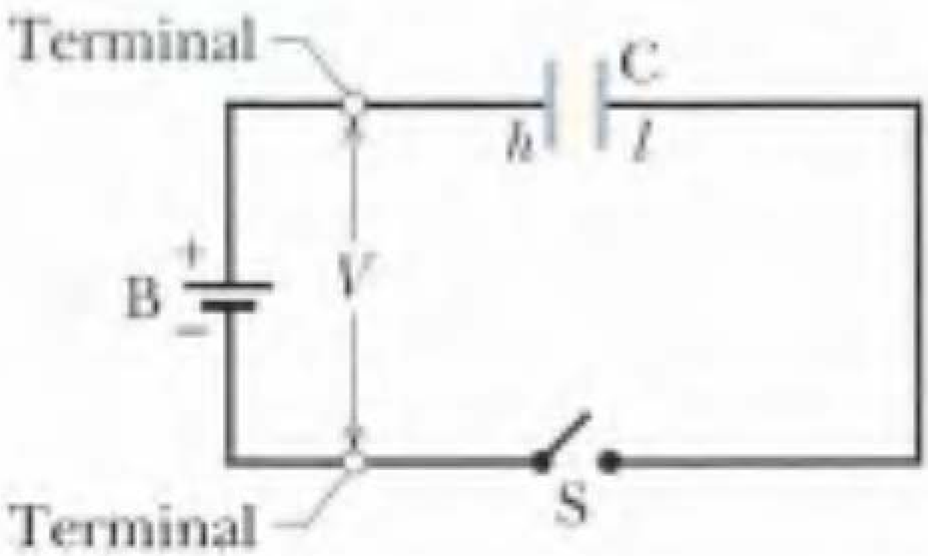
C is called the **capacitance**

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}.$$

(Farad is the unit of capacitance, however, we commonly use (pF) picofarad=10^{-12}F, (μF) microfarad=10^{-6}F, and (nF) nanofarad=10^{-9}F.)



(a)



A parallel-plate air capacitor has a capacitance of 800 pF. The charge on each plate is 6 μC. What is the potential difference between the plates ?

In this problem we just need to use the relation between the capacitance of the capacitor, its charge and the potential difference between the plates :

$$Q = CV$$

$$V = \frac{Q}{C} = \frac{6 * 10^{-6}}{800 * 10^{-12}} = 7500\ V$$

A parallel-plate air capacitor of capacitance of 100 pF has a charge of magnitude 0.1 µC on each plate. The plates are 0.5 mm apart .

(a) What is the potential difference between the plates ?

(b) What is the area of each plate ?

(c) What is the electric-field magnitude between the plates ?

(d) What is the surface charge density on each plate ?

a) To find the potential difference we need to use the equation

$$Q = CV \qquad V = \frac{Q}{C} = \frac{0.1 * 10^{-6}}{100 * 10^{-12}} = 1000\ V$$

A parallel-plate air capacitor of capacitance of 100 pF has a charge of magnitude 0.1 μC on each plate. The plates are 0.5 mm apart .

(a) What is the potential difference between the plates ?

(b) What is the area of each plate ?

(c) What is the electric-field magnitude between the plates ?

(d) What is the surface charge density on each plate ?

To find the area of the plates we need to use the expression for the capacitance :

$$C = \epsilon_0 \frac{A}{d}$$

We know d=0.5 mm=0.0005 m, then we can find the area A:

$$A = \frac{dC}{\epsilon_0} = \frac{0.0005 * 100 * 10^{-12}}{8.85 * 10^{-12}} = 0.0056\ m^2$$

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(b) What is the area of each plate ?

(c) What is the electric-field magnitude between the plates ?

(d) What is the surface charge density on each plate ?

The potential difference is related to the electric field through the following relation : $V = Ed$

$$E = \frac{V}{d} = \frac{1000}{0.0005} = 2 * 10^{6}\ V/m$$

A parallel-plate air capacitor of capacitance of 100 pF has a charge of magnitude 0.1 µC on each plate. The plates are 0.5 mm apart .

(a) What is the potential difference between the plates ?

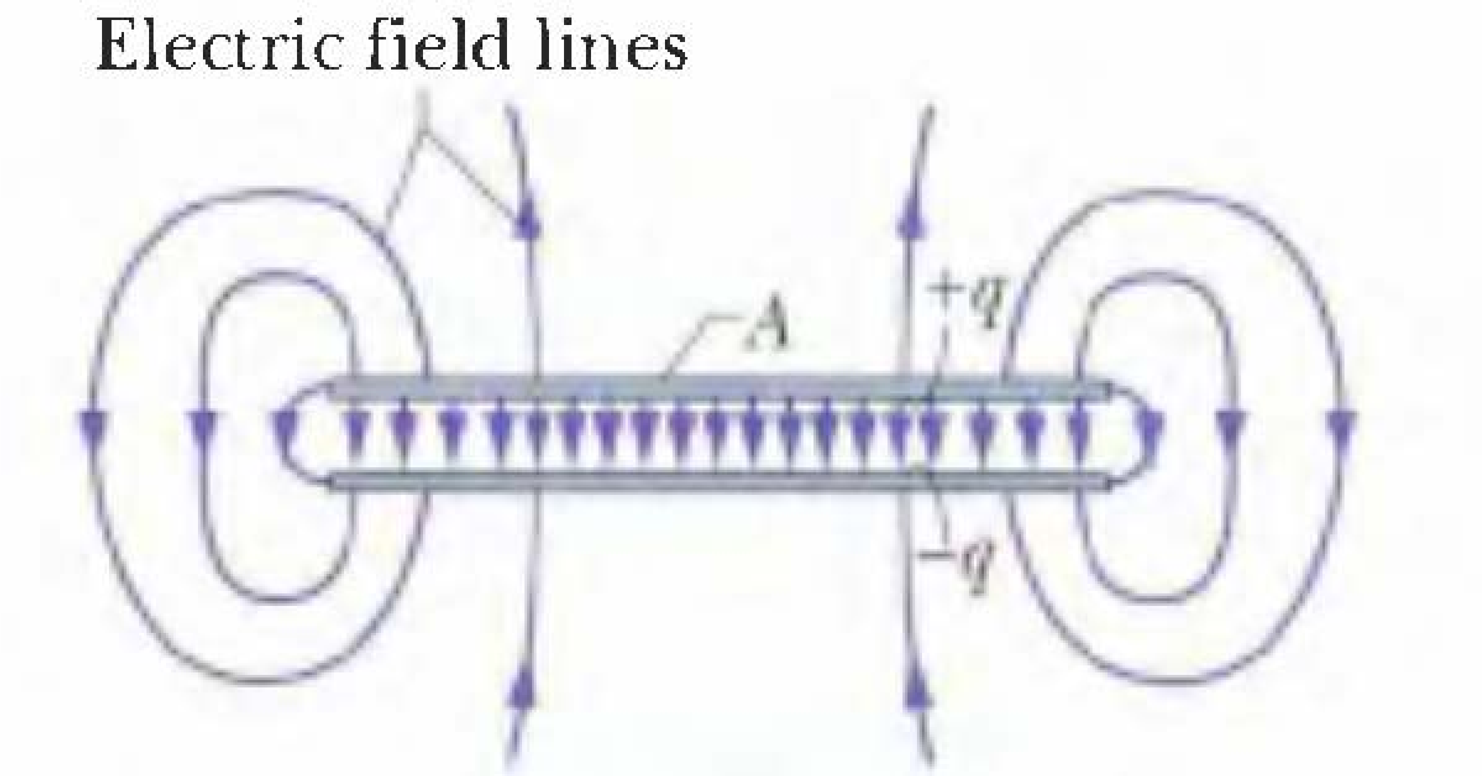
(b) What is the area of each plate ?

(c) What is the electric-field magnitude between the plates ?

(d) What is the surface charge density on each plate ?

The surface charge density can be found from the equation :

$$\sigma = \frac{Q}{A} = \frac{0.1 * 10^{-6}}{0.0056} = 1.7 * 10^{-5}\ C/m^2$$


Electric field lines
A
+q
−q

To Find Capacitance

- Assume charge, q
- Find E
- Calculate V
- Calculate C

For close parallel plates:

use Gaussian surface around positive plate

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

$$q = \varepsilon_0 E A$$

For close parallel plates:

use Gaussian surface around positive plate

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

$$V = \int_-^+ E\, ds$$

$$V = \int_-^+ E\, ds = E \int_0^d ds = Ed.$$

For close parallel plates:

use Gaussian surface around positive plate

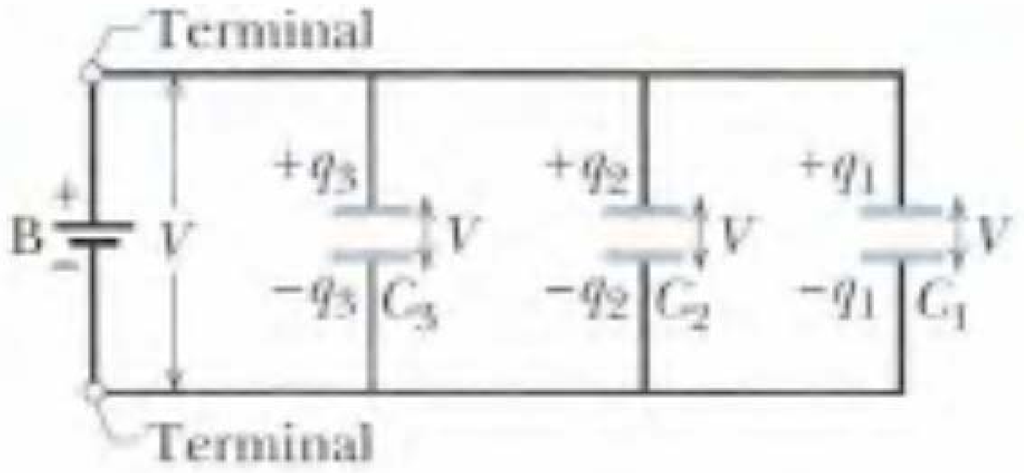
$$q = CV.$$

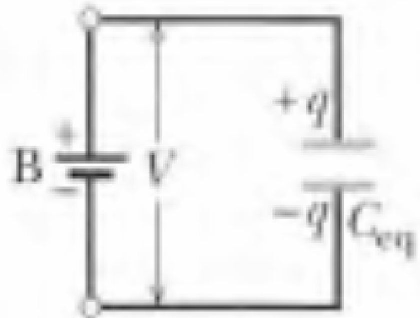
$q=CV$

$C=\epsilon_0 EA/Ed=\epsilon_0 A/d$

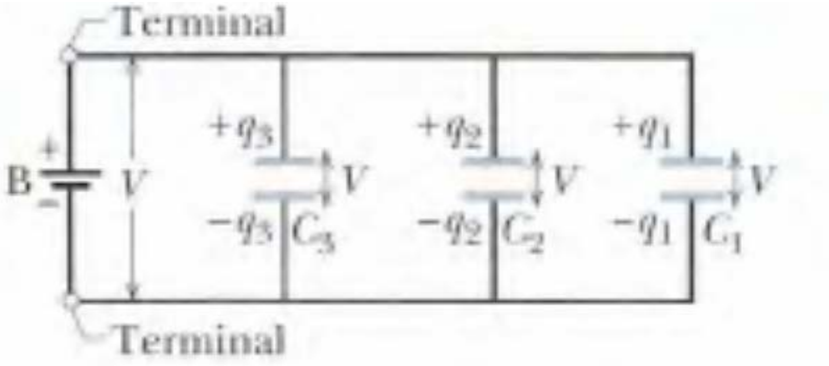
A plane-parallel capacitor has circular plates of radius $r = 10.0$ cm, separated by a distance $d = 1.00$ mm. How much charge is stored on each plate when their electric potential difference has the value $V = 100$ V? Discuss

Capacitors in Parallel





V1=V2=V3



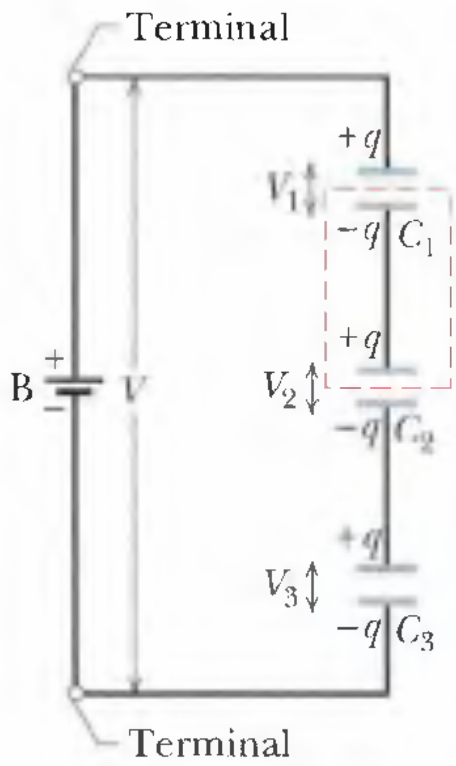
$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

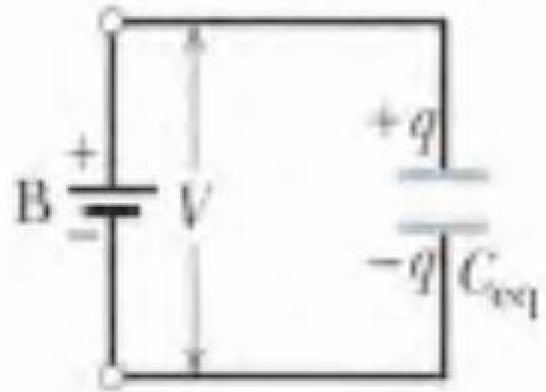
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

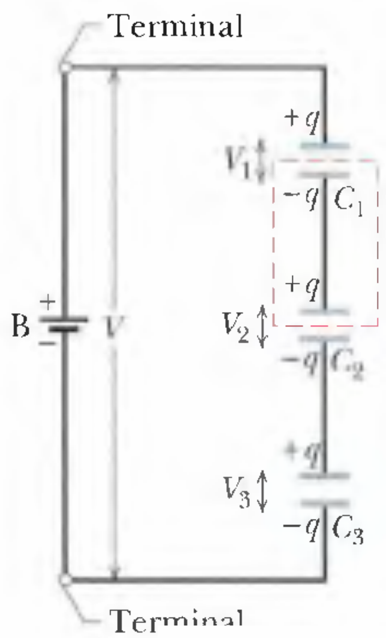
$$C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).$$

Capacitors in Series





q1=q2=q3



$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

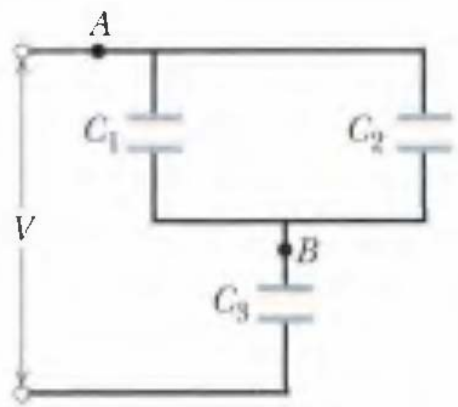
$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

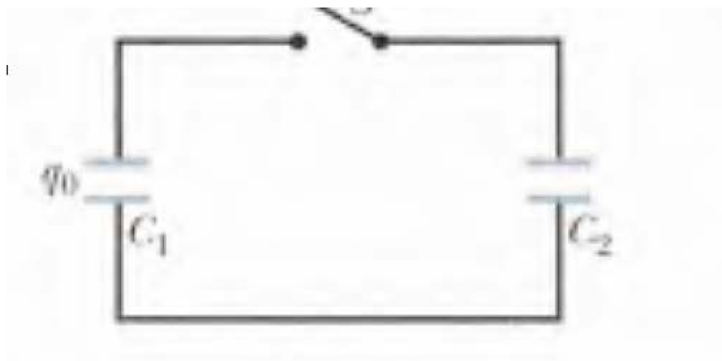
$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \qquad (n \text{ capacitors in series}).$$



$C_1 = 12.0\ \mu F$, $C_2 = 5.30\ \mu F$, and $C_3 = 4.50\ \mu F$.

Find the equivalent capacitance

(b) The potential difference applied to the input terminals in Fig. 25-10*a* is $V = 12.5$ V. What is the charge on C_1?



Capacitor 1, with $C_1 = 3.55\ \mu$F, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu$F. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

Energy Stored in an Electric Field

Reminder:

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

$$V = -\frac{W_\infty}{q}$$

The work of moving a little bit of charge from one plate to another is:

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

The work of moving all of the charge from one plate to another is:

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

The work is stored as potential energy:

$$U = \frac{q^2}{2C} \qquad \text{(potential energy).}$$

Or using q=CV can re-write as:

$$U = \tfrac{1}{2}CV^2 \qquad \text{(potential energy).}$$

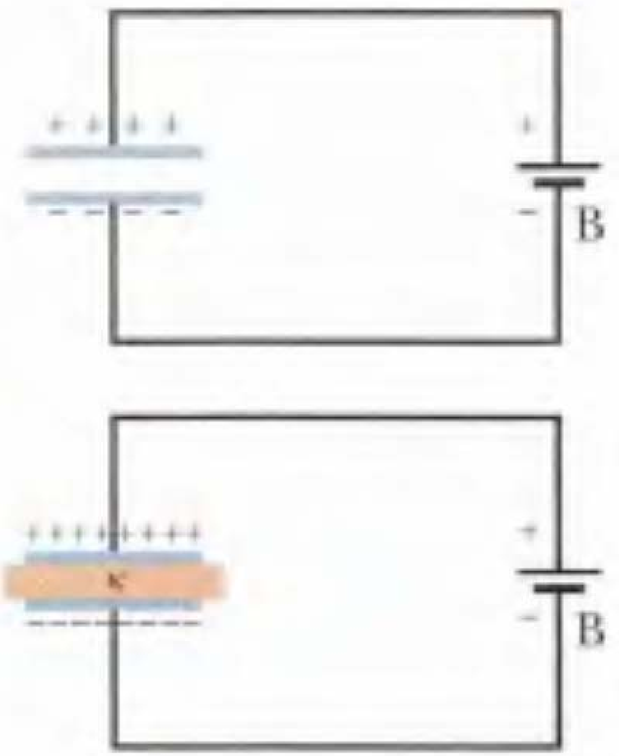
Capacitor with a Dielectric

$$C = k\,C_{air}$$

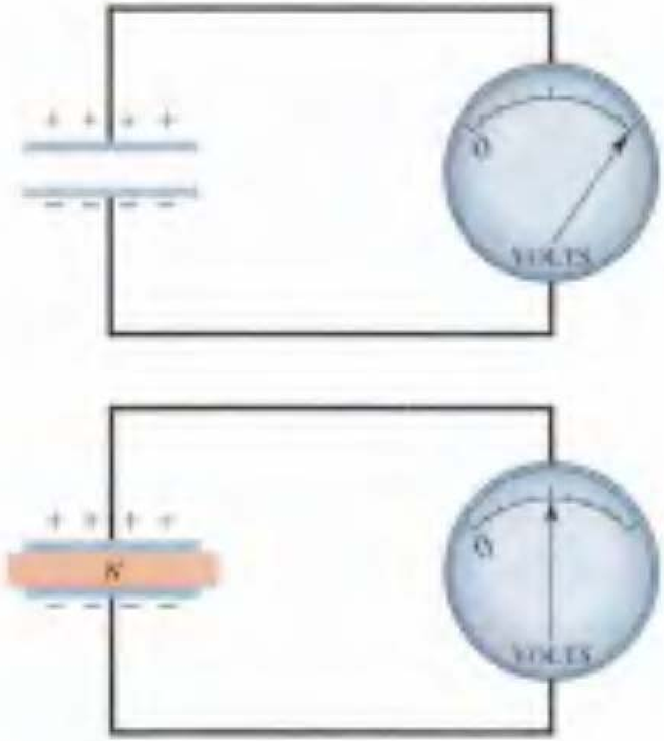
Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.



V = a constant



q = a constant

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference V = 12.5 V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

A certain parallel-plate capacitor consists of two plates, each with area 200 cm^2, separated by a 0.4-cm air gap. (*a*) Compute its capacitance. (*b*) If the capacitor is connected across a 500-V source, what are the charge on it, the energy stored in it, and the value of E between the plates? (*c*) If a liquid with $K = 2.60$ is poured between the plates so as to fill the air gap, how much additional charge will flow onto the capacitor from the 500-V source?

Working Voltage

Defined as the maximum safe potential difference beyond which there is a risk of electrical breakdown. Occurs at a fixed electrical field. For air it is 50 MV/m.

Reminder, in parallel plate:

$E=V/d$

So the smaller spaced the lower allowed voltage.

Table 23.1 Properties of Some Common Dielectrics

Dielectric Material	Dielectric Constant	Breakdown Field (MV/m)
Air	1.0006	3
Aluminum oxide	8.4	670
Glass (Pyrex)	5.6	14
Paper	3.5	14
Plexiglas	3.4	40
Polyethylene	2.3	50
Polystyrene	2.6	25
Quartz	3.8	8
Tantalum oxide	26	500
Teflon	2.1	60
Water	80	depends on time and purity

A 100-μF capacitor has a working voltage of 20 V, while a 1.0-μF capacitor is rated at 300 V. Which can store more charge? More energy?

MAKING THE CONNECTION You've got two 10-μF capacitors rated at 15 V. What are the capacitances and working voltages of their parallel and series combinations?